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Emission of ballistic acoustic phonons by quantum edge states

S M Badalian

Yerevan State University, 1 Alex Manoogian Street, 375049 Yerevan, Armenia

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Abstract. Emission of ballistic acoustic phonons by quantum edge states of one spatial quantization subband has been studied in a two-dimensional electron gas formed at the GaAs/AlGaAs heteroface. The phonon emission due to the deformation and piezoelectric interactions are considered. An analytic expression for the emission intensity distribution in phonon momenta has been derived. Detailed analysis of the intensity distribution is made in low- and high-temperature regimes. Different positions of the Fermi level are considered. It is shown that at low temperatures the phonon emission is predominantly concentrated within a narrow cone around the direction of the edge state propagation, while at high temperatures it is around the magnetic field normal to the electron plane. The emission intensity decreases when the Fermi level falls. This diminution is exponential at low temperatures.

1. Introduction

To explain the quantum Hall effect (QHE) in a two-dimensional electron gas (2DEG), since shortly after its discovery [1], the concept of quantum edge states has been advanced [2–4]. In this description the Hall conductance is determined by the number of equally populated edge states at Fermi level ε_F. When the 2DEG is subjected to a quantizing magnetic field, due to confining potential, bulk Landau levels are bent up near boundaries of the sample and intersect the ε_F, giving rise to one-dimensional transport channels. Using non-ideal probes, electrons can be selectively injected into different edge channels [5]. Recent experiments showed that such lack of equilibration between edge states is maintained for rather long macroscopic distances [6, 7] and the inter-edge-state scattering strongly depends on the channel index [8]. As a result a significant deviation from the normal QHE is to be observed (see [9]).

In such a situation to fully understand some aspects of the QHE it becomes important to study inter-edge-state scattering processes. Scattering by phonons already has been discussed in several theoretical works in different circumstances [10–15]. They all have studied the inter-edge relaxation, while it is well known that in some cases the emission and absorption of ballistic phonons by the 2DEG are more powerful tools to investigate the electron–phonon interaction [16]. Up to now such theoretical investigations under quantizing magnetic fields were limited only to considerations of bulk Landau states [17–20]. The QHE breakdown due to phonon-assisted transitions was studied and the steady state power absorption was calculated for a GaAs heterojunction [17]. The phonon emission spectrum of the heated 2DEG in a Si inversion layer was studied [18]. The role of the phonon reflection was investigated from both the Si/SiO_2 interface [19] and the GaAs/AlGaAs heteroface [20].

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Equally with the bulk Landau states, the quantum edge states also give a contribution to the emission and absorption of ballistic phonons by the 2DEG. In a recent experiment the absorption of non-equilibrium phonon pulses has been already used to investigate a backscattering of electrons between edge states on opposite sides of the sample [21]. Possibly, the absorption of ballistic phonons by edge states plays a definite role in the phonon drag effect in the QHE regime [22].

In this paper we have developed a theory of the ballistic phonon emission by quantum edge states. In the next section an analytic expression for the acoustic energy flux is derived. Different ranges of electron temperature T_e and different positions of the Fermi level ε_F are considered in section 3. In section 4 the emission due to the piezoelectric interaction is discussed. Numerical estimates are made in the last section.

2. Phonon emission: deformation potential

The spectral density of the acoustic energy flux at a point r_0 and at a frequency ω is given by

$$W_\alpha = \mathrm{Re}\left\{\frac{-\mathrm{i}\omega}{2\pi}\langle\sigma^*_{\alpha\beta}(r_0)u_\beta(r_0)\rangle_\omega\right\} \tag{1}$$

where $\sigma_{\alpha\beta}$ and u_α are the stress tensor and phonon field operators in the Heisenberg representation ($\alpha, \beta = 1, 2, 3$), $\langle\cdots\rangle_\omega$ is the Fourier transform of the correlator of the σ and u operators. To obtain W_α it is necessary to calculate the correlator of the phonon field operators $K_{\alpha\beta}(r, r') = \langle u^*_\alpha(r)u_\beta(r')\rangle_\omega$. When the 2DEG is embedded in an elastic medium and the phonon displacement is caused by the deformation electron–phonon interaction then the correlator $K_{\alpha\beta}$ is represented in the following form [20]:

$$K_{\alpha\beta}(r, r') = 2\pi\,\Xi^2\sum_{\mathrm{i,f}} f(\varepsilon_\mathrm{i})(1 - f(\varepsilon_\mathrm{f}))\,\delta(\varepsilon_\mathrm{i} - \varepsilon_\mathrm{f} - \omega)$$

$$\times\int \mathrm{d}r_1\int \mathrm{d}r_2\,\Psi^*_\mathrm{i}(r_1)\Psi_\mathrm{f}(r_1)\Psi_\mathrm{i}(r_2)\Psi^*_\mathrm{f}(r_2)D^*_\alpha(r, r_1|\omega)D_\beta(r', r_2|\omega) \tag{2}$$

in which

$$D_\alpha(r, r'|\omega) = \partial'_\alpha G_{\alpha\beta}(r, r'|\omega). \tag{3}$$

Here Ξ is the deformation potential constant, f is the Fermi factor and $\mathbf{G}$ means the Green function of the elasticity theory. The initial (final) energy and wave function of electrons are $\varepsilon_{\mathrm{i(f)}}$ and $\Psi_{\mathrm{i(f)}}$. The 2DEG is subjected to a quantizing magnetic field normal to the plane of electrons, $\boldsymbol{B}||z$ so that

$$\begin{aligned}\Psi(r) &= \exp(\mathrm{i}kx)\chi_{lk}(y)\psi_n(z)\\ \varepsilon &= \varepsilon_l(k) + \varepsilon_n.\end{aligned} \tag{4}$$

Here $\varepsilon_l(k)$ and χ_{lk} are the energy and wave function of the quantum edge state specified by the Landau index l and momentum k (figure 1). The energy and wave function of the spatial quantization of the 2DEG are ε_n and ψ_n. We shall assume that all electrons occupy a single level n and the electron transitions take place between edge states with different l and k. Usually the 2DEG is located near interfaces separating different elastic materials. However, if we do not treat the reflection of phonons from these interfaces and use for $\mathbf{G}$ the Green function of the bulk elastic medium, we may find for the kernel of the correlator (2)

$$D^*_\alpha(r_0, r_1|\omega)D_\alpha(r_0, r_2|\omega) = \frac{\omega^2}{16\pi^2 r_0^2\rho^2 s^6}\exp[-\mathrm{i}q\cdot(r_2 - r_1)] \tag{5}$$

where ρ is the mass density of the elastic medium, s and $q = n\omega/s$ are the velocity and momentum of LA phonons. From the experimental situation it follows that the energy flux emitted from the 2DEG, located near one side of the sample, is detected in an infinitely distant point on the other side of the sample. Therefore, in obtaining the kernel (5) we have assumed $r_0 = n\infty$; n is a unit vector towards the detector. Substituting equation (5) into equation (2) we obtain the acoustic energy flux density in a frequency range $d\omega$ emitted from a unit length (towards x) of the 2DEG into a solid angle do around the direction of n

$$W^{\mathrm{DA}}_{l\to l'}(q) = \frac{\Xi^2\omega^4}{8\pi^3\rho s^5 \delta v_{ll'}} f(\varepsilon_l(k_0))(1 - f(\varepsilon_{l'}(k_0 + q_x)))|Q_{ll'}(q_x, q_y)|^2 |I_{00}(q_z)|^2 \tag{6}$$

where the form factors in directions y and z are

$$Q_{ll'}(q_x, q_y) = \int \mathrm{d}y\ \chi_{lk_0}(y) \mathrm{e}^{-\mathrm{i}q_y y} \chi_{l'k_0+q_x}(y) \tag{7}$$

and

$$I_{nn}(q_z) = \int \mathrm{d}z\ |\psi_n(z)|^2 \mathrm{e}^{-\mathrm{i}q_z z} \tag{8}$$

In equation (6) $\delta v_{ll'} = |v_l(k_0) - v_{l'}(k_0 + q_x)|$, $v_l(k)$ is the group velocity of the edge state l with the momentum k, $k_0 = k_0(q)$ is determined from the energy and momentum (in the x direction) conservation laws, i.e. from the following equation:

$$\varepsilon_l(k) - \varepsilon_{l'}(k + q_x) = \omega = s\sqrt{q_x^2 + q_y^2 + q_z^2}. \tag{9}$$

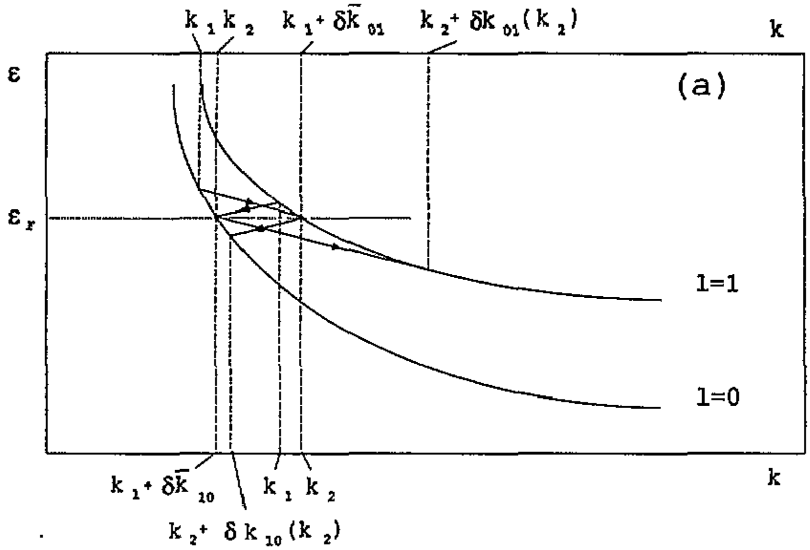
Actually, despite the lack of momentum conservation in y and z directions, the phonon momentum q uniquely determines the initial $l, k_0(q)$ and final $l', k_0(q) + q_x$ edge states in a phonon emission. Thus equation (6) gives the distribution of the emission intensity in phonon momenta.

3. Low- and high-temperature regimes

In this section we will discuss the situation in low- and high-temperature regimes in which the emission is qualitatively different. From equation (9) one may see that for a given k, q_x cannot be less than some value $\delta k_{ll'}(k)$ determined from equation (9) at $q_y = q_z = 0$. Hence, only phonons with frequencies $\omega \geqslant s\,\delta k_{ll'}(k)$ can be emitted from the edge state l, k. On the other hand, the effectiveness of each emission act depends on the position of the Fermi level and electron temperature. Due to the deficit of the hot electrons with energies above $\varepsilon_l(k_1) = \varepsilon_{\mathrm{F}} + s\,\delta k_{ll'}(k_1)$ and deficit of the free final states with energies below $\varepsilon_{l'}(k_2 + s\,\delta k_{ll'}(k_2)) = \varepsilon_{\mathrm{F}} - s\,\delta k_{ll'}(k_2)$, the emission acts from states l, k with k between k_1 and k_2 are comparatively more efficient. The energy spectrum of the edge states is arranged so that for a fixed energy level we always have $v_l > v_{l'}$ if $l < l'$. Therefore $\delta k_{ll'}(k)$ achieves its minimum $\delta\bar{k}_{ll'} = \delta k_{ll'}(k_1)$ at the upper edge of the interval (k_1, k_2) (figure 1(a)). Thus from the Fermi factors and form factors (7), (8) we have the following obvious restrictions on the emission processes:

$$|q_x - \delta\bar{k}_{ll'}| \leqslant T_{\mathrm{e}}/s \qquad |q_y| \leqslant \min\{a_B^{-1}, T_{\mathrm{e}}/s\} \qquad |q_z| \leqslant \min\{d^{-1}, T_{\mathrm{e}}/s\} \tag{10}$$

where a_B is the magnetic length, d is the characteristic length of electron motion in the z direction. (As long as the electron transitions take place between edge states of one spatial quantization, d is the minimum length scale of the problem.) Here q_x is not a free parameter but it should firstly satisfy the momentum conservation in the x direction.



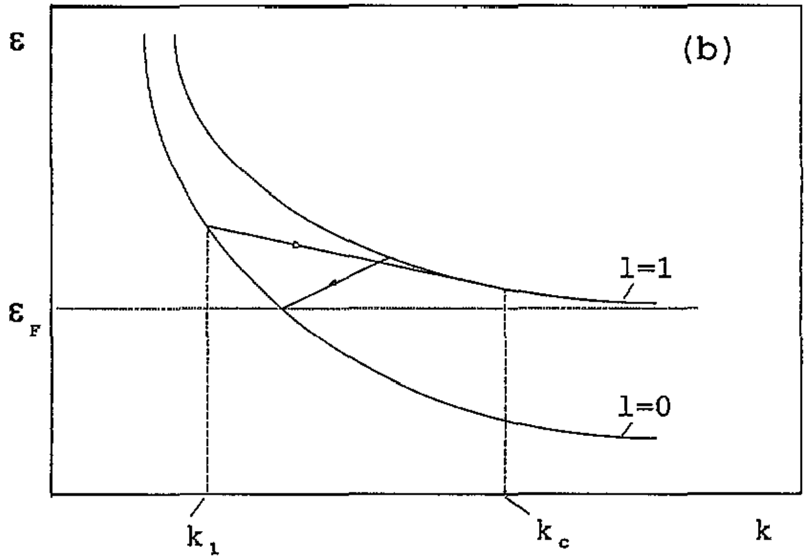


Figure 1. A schematic diagram of the edge state energy spectrum. (a) Transitions $0 \to 1$ and $1 \to 0$ are shown. At low temperatures, states close to the state $0k_1$ are important. (b) If the Fermi level is close to the bulk Landau level transitions $0 \to 1$ are possible involving states above the threshold k_c. At high temperatures states disposed at a separation of T_e above and below ε_F are important.

At low temperatures $T_e \ll s\,\delta\bar{k}_{ll'}$ we have $q_x \approx \delta\bar{k}_{ll'}$, $q_z \sim T_e/s$. If the confining potential is not smooth, $\delta\bar{k}_{ll'} \sim a_B \gg T_e/s$, so that $q_y \sim T_e/s$ and $q_x \gg q_y$. In the case of the smooth potential $\delta\bar{k}_{ll'} \gg a_B^{-1}$ so that T_e/s and a_B^{-1} may be of the same order of magnitude. But in any case, the relation $q_x \gg q_y$ remains true. Thus at low temperatures $q_x \gg q_y, q_z$, i.e. phonons are mainly emitted in the x direction. Therefore one may substitute $q_y = q_z = 0$ and $q_x = \delta\bar{k}_{ll'}$ into the prefactor of (6) and form factors (7), (8). Taking into account that $I_{nn}(0) = 1$, one may find for the acoustic energy flux distribution space

$$W^{\mathrm{DA}}_{l\to l'}(q) = \frac{\Xi^2\,\delta\bar{k}^4_{ll'}}{8\pi^3\rho s\,\delta\bar{v}_{ll'}}\,Q^2_{ll'}(q_x,0)\exp\left\{-\frac{s\delta\bar{k}_{ll'}}{T_e} - \frac{s}{T_e}\left[(q_x - \delta\bar{k}_{ll'}) + \frac{q_y^2+q_z^2}{2\,\delta\bar{k}_{ll'}}\right]\right\} \tag{11}$$

where $\delta\bar{v}_{ll'} = \delta v_{ll'}$ at $k = k_1$ and $q_x = \delta\bar{k}_{ll'}$. In the case of the non-smooth potential we have only one length scale a_B in the (x, y) plane and so $Q_{ll'} \sim 1$ if $q_y \sim a_B^{-1}$. In the case of the smooth potential $Q_{ll'}$ may be calculated explicitly [13]

$$Q^2_{ll'}(q_x,0) = (2^{l+l'}l!l'!)^{-1}(q_x a_B)^{2l+2l'}\exp\{-(q_x a_B)^2/2\}. \tag{12}$$

Now one may see that at low temperatures electrons in edge states emit almost monochromatic acoustic phonons with frequencies $\omega \approx s\,\delta\bar{k}_{ll'}$. The emission is predominantly concentrated within a narrow cone around the edge state propagation. The emission intensity exponentially drops out of the cone. For the non-smooth potential, the emission cone is isotropic in the (y, z) plane and the cone angle is determined by $\sqrt{\delta\bar{k}_{ll'}T_e/s}$, while in the case of the smooth potential by $\max\{a_B^{-1}, \sqrt{\delta\bar{k}_{ll'}T_e/s}\}$ in the y direction and by $\sqrt{\delta\bar{k}_{ll'}T_e/s}$ in the z direction. A momentum spread in the x direction is $\Delta q_x \equiv q_x - \delta\bar{k}_{ll'} \sim T_e/s$. It should be noted that only some of the states from the interval between k_1 and k_2 give an essential contribution to the phonon emission processes. When k varies in this interval, $\delta k_{ll'}(k)$ is changed by $\delta\bar{k}_{ll'}\,s\,\delta\bar{v}_{ll'}/v_l v_{l'}$ which is much less than $\delta\bar{k}_{ll'}$ even if $\delta\bar{v}_{ll'} \sim v_l, v_{l'}$. However, only states l, k for which $s\,\delta k_{ll'}(k)\,s\,\delta\bar{v}_{ll'}/v_l v_{l'} \sim T_e$ are effective in the emission processes. Because of the velocity asymmetry $v_l > v_{l'}$ if $l < l'$, the number of these states decreases when the Fermi level falls. Simultaneously $\delta\bar{k}_{ll'}$ increases which leads to the suppression of the phonon emission (cf. [8]). If the Fermi level is very close to the bulk Landau level (figure 1(b)), $v_{l'} \sim s$ and $\delta v_{ll'} \sim v_l$ so that $s\,\delta k_{ll'}(k)$ is changed by $s\,\delta\bar{k}_{ll'}\,s\,\delta\bar{v}_{ll'}/v_l v_{l'} \sim s\,\delta\bar{k}_{ll'} \gg T_e$ in the interval (k_1, k_2). Therefore the main contribution to the phonon emission comes from the single edge state l, k_c where k_c is defined from $v_{l'}(k_c) + \delta k_{ll'}(k_c) = s$. The emission processes from states lying below k_c are forbidden by conservation laws, while those from states above k_c are forbidden due to the strong deficit of the hot electrons. Such a critical point k_c exists only for transitions $l \to l'$ with $l < l'$. In the case of $l > l'$ there is no critical point in the edge state spectrum and a smooth transition is possible to the case of the phonon emission from the bulk Landau levels [20]. Because $\delta\bar{k}_{ll'}$ for $l > l'$ is always more than for $l < l'$, there is an asymmetry between emission processes $l \to l'$ and $l' \to l$. This asymmetry depends on the Fermi level position and is pronounced when the Fermi level is close to the bulk Landau level: $W_{l\to l'} \gg W_{l'\to l}$ if $l > l'$.

At high temperatures $T_e \gg s\,\delta\bar{k}_{ll'}$ the states which are more efficient in the emission processes are disposed above and below the Fermi level at separation of the order of T_e. Therefore it is clear that $q_x \sim \delta\bar{k}_{ll'}$ and $q_y \sim a_B^{-1}$ are determined, respectively, only by the momentum conservation and by the magnetic length according to equation (10). Correlation between q_x and q_y is determined by the shape of the confining potential just as at the low temperatures. In the z direction we have $q_z \sim \min\{d^{-1}, T_e/s\}$ so that the relation $q_z \gg q_x, q_y$ holds for any d and T_e as well as for any shape of the confining potential. Thus in contrast to the low-temperature regime, at high temperatures the phonon emission is concentrated within a narrow cone around the magnetic field normal to the 2DEG plane. Using the variation wave function for the lowest subband $n = 0$ we have $|I_{00}|^2 = [1 + (q_z d)^2]^{-3}$. Taking into account that in this regime $f(\varepsilon_l(k_0))(1 - f(\varepsilon_{l'}(k_0 + q_x))) \sim \exp\{-sq_z/T_e\}$ we obtain for the emission intensity

$$W^{\mathrm{DA}}_{l\to l'} = \frac{\Xi^2 {q_z}^4}{8\pi^3\rho s\,\delta\bar{v}_{ll'}}\frac{Q^2_{ll'}(q_x,0)}{[1+(q_z d)^2]^3}\exp\left\{-\frac{sq_z}{T_e}\right\}. \tag{13}$$

By comparing equations (11) and (13) one may see that at low temperatures the phonon emission is exponentially suppressed. At high temperatures electrons emit phonons with frequencies $\omega \sim T_e$ and the emission is much more intense than at low temperatures when $\omega \sim s\,\delta\bar{k}_{ll'} \gg T_e$. In the high-temperature regime the phonon emission is not so sensitive to the Fermi level position.

4. Phonon emission: piezoelectric potential

Up to now we have considered only the deformation electron–phonon interaction. Direct calculation shows that to find the phonon emission intensity due to the piezoelectric coupling the following replacement should be made [23] in equation (6): $\Xi^2(\omega^2/s^2) \to (e\beta)^2$ where β is the piezoelectric modulus of the crystal averaged over directions of a phonon's propagation and its polarizations. Therefore, taking the values of Ξ and β from [24], for GaAs we find

$$\frac{W^{\mathrm{DA}}}{W^{\mathrm{PA}}} = \frac{\Xi^2(\omega^2/s^2)}{(e\beta)^2} = \left(5.6\frac{\omega}{s}\ (\mathrm{nm}^{-1})\right)^2 . \tag{14}$$

At low temperatures $\omega \sim s\,\delta\bar{k}_{ll'}$. Therefore for the non-smooth confining potential

$$\frac{W^{\mathrm{DA}}}{W^{\mathrm{PA}}} \sim \left(\frac{5.6}{a_B\,(\mathrm{nm})}\right)^2 = \frac{B\ (\mathrm{T})}{20.9} \tag{15}$$

and DA interaction is suppressed with respect to PA interaction for not so high magnetic fields. In the case of the smooth potential

$$\frac{W^{\mathrm{DA}}}{W^{\mathrm{PA}}} \sim \left(\frac{\delta\bar{k}_{ll'}\,a_B}{4.6}\right)^2 B\ (\mathrm{T}) \tag{16}$$

and because $\delta\bar{k}_{ll'}\,a_B \gg 1$, even at $B \sim 1$ T, the DA and PA interaction give roughly the same contribution to the phonon emission.

At high temperatures we have $\omega \sim T_e$ so that

$$\frac{W^{\mathrm{DA}}}{W^{\mathrm{PA}}} \sim \left(\frac{T_e\ (\mathrm{K})}{6.9}\right)^2 \tag{17}$$

i.e. at actual temperatures $T_e \sim 10$ K, contributions of the DA and PA interaction are approximately of the same order of magnitude.

To compare the contributions of edge and bulk Landau states to the phonon emission from the 2DEG one has to average equation (6) in k_0 and to make a substitution $\pi\delta(\cdots) \to \tau$ in equation (8) of [20] where τ determines the Landau level broadening†. In this way, in the high-temperature regime one may obtain for magnetic fields $\omega_B \sim T_e$ (ω_B is the cyclotron energy) and for $\delta\bar{v}_{ll'} \sim v_l, v_{l'}$, even in heterojunctions of such high quality with mobility $\mu = 10^5\ \mathrm{cm^2\ V^{-1}\ s^{-1}}$, the contribution of the edge states to the cooling of the 2DEG at least is not less than the contribution of the bulk Landau states. In the regime of low temperatures, because $\omega_B \gg s\,\delta\bar{k}_{ll'} \gg T_e$, the phonon emission is only due to inter-edge-state transitions while the emission is practically absent at electron transitions between bulk Landau states.

† The substitution $\pi\delta(\cdots) \to 1$ made in [20] is incorrect. This changes only absolute values of the emission intensities of LA, TA and SA phonons while conclusions reached in [20] about the heteroface effect on the phonon emission remain true.

5. Numerical estimates

Finally we estimate the acoustic energy flux emitted by edge states for the case of the non-smooth confining potential. In the low-temperature regime the emission goes mainly via piezoelectric coupling. At the peak of the emission $q_x = \delta\bar{k}_{ll'}$ and $q_y = q_z = 0$, the emission intensity may be represented in the form

$$W_{l\to l'}^{\mathrm{PA}} = \frac{1}{(2\pi)^2}\frac{ms}{\bar{\tau}_{\mathrm{PA}}}\frac{v_B}{\delta\bar{v}_{ll'}}\left(\frac{\delta\bar{k}_{ll'}}{p_B}\right)^2 \exp\left(-\frac{s\,\delta\bar{k}_{ll'}}{T_e}\right) \tag{18}$$

where the nominal time of the interaction is defined as

$$\frac{1}{\bar{\tau}_{\mathrm{PA}}} = \frac{(e\beta)^2 p_B}{2\pi\hbar\rho s^2} \tag{19}$$

and $p_B = a_B^{-1} = mv_B$ is the magnetic momentum. For the GaAs we have $ms = 3.1 \times 10^{-28}$ J s m^{-1} and at $B = 2$ T, $\tau_{\mathrm{PA}} = 36.4$ ps. Taking $\delta\bar{v}_{10} = v_B$ and $\delta\bar{k}_{10} = p_B$ we have $s\,\delta\bar{k}_{10} = 2.2$ K for $B = 2$ T so that at $T_e = 0.5$ K we obtain $W_{1\to 0}^{\mathrm{PA}} = 3.1 \times 10^{-21}$ W s m^{-1}. For the emission cone angle we find $\theta = \tan^{-1} q_x/\sqrt{q_y^2 + q_z^2} = 25°$.

At high temperatures $W_{l\to l'}^{\mathrm{DA}}$ and $W_{l\to l'}^{\mathrm{PA}}$ may be represented in the forms

$$W_{l\to l'}^{\mathrm{DA}} = \frac{1}{(2\pi)^2}\frac{ms}{\bar{\tau}_{\mathrm{DA}}}\frac{v_B}{\delta\bar{v}_{ll'}}\left(\frac{T_e}{sp_B}\right)^4 \frac{x^4 e^{-x}}{[1+\eta^2x^2]^3} \tag{20}$$

where the nominal time of the interaction is defined as

$$\frac{1}{\bar{\tau}_{\mathrm{DA}}} = \frac{\Xi^2 p_B^3}{2\pi\hbar\rho s^2} \tag{21}$$

and

$$W_{l\to l'}^{\mathrm{PA}} = \frac{1}{(2\pi)^2}\frac{ms}{\bar{\tau}_{\mathrm{PA}}}\frac{v_B}{\delta\bar{v}_{ll'}}\left(\frac{T_e}{sp_B}\right)^2 \frac{x^2 e^{-x}}{[1+\eta^2x^2]^3}. \tag{22}$$

Here $x = sq_z/T_e$ and $\eta = T_e/(s/d)$. The emission peaks are defined from conditions that the last factor in equations (20) and (22) should be maximum. For the GaAs/AlGaAs heterojunction $d = 3$ nm, $s = 5 \times 10^3$ m s^{-1} and so $\hbar s/d = 13$ K. Taking $T = 10$ K we obtain that the emission peaks for DA and PA interaction are determined, respectively, from $x = 1.21$ and $x = 0.85$. This means that at the emission peak, frequencies of phonons emitted due to the deformation coupling are approximately 1.4 times larger than frequencies of phonons emitted due to the piezoelectric coupling. At $B = 2$ T we have $\bar{\tau}_{\mathrm{DA}} = 382$ ps so that taking again $\delta\bar{v}_{10} = v_B$ we obtain $W_{1\to 0}^{\mathrm{DA}} = 1.03 \times 10^{-18}$ W s m^{-1} and $W_{1\to 0}^{\mathrm{PA}} = 0.53 \times 10^{-18}$ W s m^{-1}. For the emission cone angle we find $\theta = \tan^{-1} q_z/\sqrt{q_x^2 + q_y^2} = 10°$ (for DA) and $\theta = 14°$ (for PA).

It should be observed that in the smooth confining potential the phonon emission is suppressed exponentially. At low temperatures suppression takes place for two reasons: firstly, because of the threshold nature of the emission, electrons are forced to emit phonons with larger frequencies $\omega \sim s\,\delta\bar{k}_{ll'} \gg sp_B$ and secondly, because of the exponential smallness of the overlap integral $Q_{ll'}$, while at high temperatures suppression takes place only for the last reason.

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References

[1] von Klitzing K, Dorda G and Pepper M 1980 *Phys. Rev. Lett.* **45** 494
[2] Laughlin R B 1981 *Phys. Rev.* B **23** 5632
[3] Halperin B I 1982 *Phys. Rev.* B **25** 2185
[4] MacDonald A H and Streda P 1984 *Phys. Rev.* B **29** 1616
[5] Büttiker M 1988 *Phys. Rev.* B **38** 9375
[6] van Wees B J, Willems E M M, Harmans C J P M, Beenakker C W J, van Houten H, Williamson J G, Foxon C T and Haris J J 1989 *Phys. Rev. Lett.* **62** 1181
[7] Komiyama S, Hirai H, Sasa S and Hiyamizu S 1989 *Phys. Rev.* B **40** 12 566
[8] Alphenaar B W, McEuen P L and Wheeler R G, 1990 *Phys. Rev. Lett.* **64** 677
[9] Haug R J 1993 *Semicond. Sci. Technol.* **8** 131
[10] Martin T and Feng S 1990 *Phys. Rev. Lett.* **64** 1971
[11] Martin T and Feng S 1991 *Phys. Rev.* B **44** 9084
[12] Khaetskii A V and Matveev K A 1992 *J. Phys.: Condens. Matter* **4** 3491
[13] Komiyama S, Hirai H, Ohsawa M, Matsuda Y, Sasa S and Fujii T 1992 *Phys. Rev.* B **45** 11 085
[14] Badalian S M, Levinson Y B and Maslov D L 1991 *JETP Lett.* **53** 595
[15] Maslov D L, Levinson Y B and Badalian S M 1992 *Phys. Rev.* B **46** 7002
[16] Challis L J, Toombs G A and Sheard F W 1987 *Physics of Phonons* ed T Paszkiewicz (Berlin: Springer) p 348
[17] Heinonen O, Taylor P L and Girvin S M 1984 *Phys. Rev.* B **30** 3016
[18] Toombs G A, Sheard F W, Neilson D and Challis L J 1987 *Solid State Commun.* **64** 577
[19] Kitagawa H and Tamura S 1992 *Phys. Rev.* B **46** 4277
[20] Badalian S M and Levinson Y B 1991 *Phys. Lett.* **155A** 200; 1992 *Phys. Lett.* **170A** 229
[21] Kent A J, McKitterick D J, Challis L J, Hawker P, Mellor C J and Henini M 1992 *Phys. Rev. Lett.* **69** 1684
[22] Dietzel F, Dietsche W and Ploog K to be published
[23] Badalian S M, Rößler U and Potemski M 1993 *J. Phys.: Condens. Matter* **10** 2309
[24] Gantmakher V F and Levinson Y B 1987 *Carrier Scattering in Metals and Semiconductors* (Amsterdam: North-Holland)